Analytical Study of the Characteristic Parameters of Microstripline Coupler and Their Variation with Geometry and Frequency Using Different Substrates

Surya Deo Choudhary, Rajeev Kumar, Md. Irshad Alam, Dr. R.K. Singh

Abstract- This paper presents the Analytical study of characteristic parameters of the microstripline coupler and their variation with strip geometries, substrate permittivity and frequency using different substrates. Respective tables and graphs shows the dependence of characteristic impedance, phase velocity and guide wavelength on stripwidth of the metal, on spacing between two metal strips, on dielectric constant of the substrate material for both the even and odd-modes. Mathematical formulation method is used which is based on the conformal transformation technique developed by H.A. Wheeler and calculation is based on computer programming developed by S.K. Kaul using closed form formula of Schwartzman.

Index Terms: Microstripline, Coupler, Substrate, Frequency, Impedance, Phase Velocity, Even & Odd-modes, and Conformal Transformation.

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1. INTRODUCTION

Several empirical expressions useful models have been proposed for microstripline coupler. This chapter deals with the Analytical Studies of the characteristic parameters of microstripline couplers and their variation with geometry and frequency using different substrates for the calculation of different types of losses and thermal effects produced in the coupled microstriplines when the waves propagate through the structure. Here the mathematical formulation method is used which is based on the conformal transformation technique developed by H.A. Wheeler and calculation is based on computer programming developed by S.K. Kaul using closed form formula of Schwartzman.
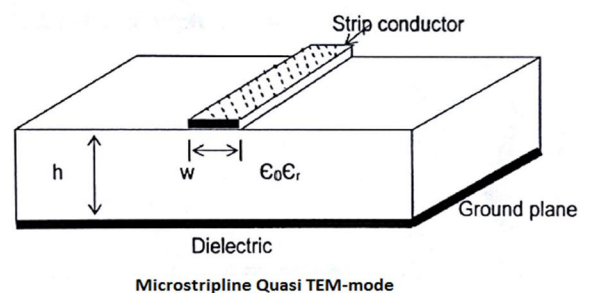
2. AIMS OF VARIOUS COMPUTATIONS

The present work involves the problems in Quasi-Static limit in lower Giga-Hertz range of frequency. The quasi-TEM allows the magnetic and electric fields to be considered, separately in case of even and odd-modes of wave propagation.
When only the magnetic field is considered, the dielectric inhomogeneity is ignored, since the dielectric medium is treated as free space.

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But when considering the electric field the inhomogeneity must be taken into account, since the normal component of electric field is discontinuous at the dielectric interface.

3. FORMULATION OF CHARACTERISTIC IMPEDANCES FOR EVEN & ODD–MODES OF A MICROSTRIPLINE COUPLER

When a microstripline is placed parallel to another microstripline, coupler is formed. The power flowing in one line is coupled to the other line either in same direction or in opposite direction. The power flowing in same direction is referred to as even-mode of propagation and in opposite direction is referred to as an odd-mode of propagation.

Microstripline Quasi TEM-mode

The characteristic impedance in case of both even and odd-modes can be calculated with the help of elementary transmission line equation expressed as:

$$Z_O = 1/V_P C_P \quad \text{----------------------------}(1)$$

Where, V_P = phase velocity of the wave travelling along the microstripline.

C_P = capacitance per unit length of the line.

Capacitance C_P of the isolated microstrip structure is expressed as:

$$C_P = C_{PP} + C_{PPU} + C_f$$

$$= (\epsilon_{reff}/c\eta)(w/n) + (2/3)(\epsilon_{reff}/c\eta)(w/h) + (\epsilon_{reff}/c\eta)(2.7/log(4h/t)) \quad \text{------------------(2)}$$

Where, C_{PP} = parallel plate capacitance between lower surface of the microstrip and the ground plane.

C_{PPU} = capacitance between the upper surface of the microstrip and the ground plane.

C_f = the fringing capacitance at the edges of the microstrip.

w = microstrip width.

ϵ_{reff} = effective dielectric constant of the medium.

h = height of the substrate.

η = free space impedance = 377 Ω

c = velocity of light in free space = 3.0 x 108 m/s.

t = microstrip thickness.

The phase velocity V_P can be calculated by the formula,

$$V_P = c/\sqrt{\epsilon_{reff}} \quad \text{-----------------------------(3)}$$

For wide strip, $\epsilon_{reff} = \epsilon_r$

For narrow strip, $\epsilon_{reff} = (\epsilon_r + 1)/2$

Where, ϵ_r = relative dielectric constant.

From equations (1), (2) and (3), we get,

$$Z_O = (\eta/\sqrt{\epsilon_{reff}})[1/\{w/h + 2w/3h + (2.7/log(4h/t))\}] \quad \text{---------------------------------------(4)}$$

On the basis of this expression the calculations give the characteristic impedance, the propagation constant and other transmission parameters of a single microstrip structure.

In even-mode the electric field lines follow the pattern fairly similar to that of the isolated conductor. In case of odd-mode, the two conductors are linked by the electric field lines.

Equation (4) is useful in calculating characteristic impedance of microstrip coupler in even and odd-mode by replacing C_P with C_{Pe} and C_{Po} for even and odd-mode

respectively. VP is replaced by V_{Pe} and V_{Po}. ϵ_{reff} is replace d by $(\epsilon_{reff})_e$ and $(\epsilon_{reff})_o$ and Z_O is replaced by Z_{oe} and Z_{oo} respectively for even and odd-modes.

3.1 EVEN-MODE CHARACTERISTIC IMPEDANCE (Z_{oe})

The total capacitance for even-mode coupled lines is expressed as:

$$C_{Pe} = C_{PPe} + ½ C_{PPU} + ½ C_f + ½ C'_{PPU} + ½ C'_f$$

Where, C_{PPe} = parallel plate capacitance as in eqn. (2) for even-mode.

C_{PPU} = capacitance between upper surface of the conductor and ground plane as in eqn.(2).

C'_{PPU} = capacitance between strip conductor and ground plane enclosed b/w two striplines.

$$= (\epsilon_{reff}/3c\,\eta)[w/h(w/s + 1)]$$

C_f = fringe capacitance at the edge of the striplines as in equation (2).

C'_f = fringe capacitance between two edges of the microstripline.

$$= (\epsilon_{reff}/c\,\eta)(2.7/log(4h/t))(1/((w/s) + 1)$$

Now we can write the characteristic impedance for even-mode configuration as:

$$Z_{Oe} = (\eta/\sqrt{\epsilon_{reff}}) [1/(w/h) + (w/3h) + (1.35/log(4h/t) + (w/3h) (1/(w/s) + 1)) + (1.35/log(4h/t) (1/((w/s) + 1))]$$

For t = 0,

$$Z_{Oe} = (\eta/\sqrt{\epsilon_{reff}}) [1/\{(w/h)[1 + (1/3\sqrt{\epsilon_{reff}})] + (1/3\sqrt{\epsilon_{reff}}) (1/(w/s) + 1)\}]$$

3.2 ODD-MODE CHARACTERISTIC IMPEDANCE (Z_{OO})

In case of odd-modes coupled lines, the total capacitance (C_{Po}) is determined in terms of the following components:-

C''_{PPU} = capacitance between strip conductor and ground plane enclosed b/w two striplines.

$$= (8/3)(\sqrt{\epsilon_{reff}}/c\,\eta)$$

C''_f = fringe capacitance between two edges of the microstripline.

$$= (\epsilon_{reff}/c\,\eta)[2.7/log(4h/s)/\textstyle\prod t)$$

The total capacitance of the odd-mode coupled lines is expressed as:

$$C_{Po} = C_{PP} + \tfrac{1}{2} C_{PPU} + \tfrac{1}{2} C_f + \tfrac{1}{2} C''_{PPU} + \tfrac{1}{2} C''_f$$

Now we can write the characteristic impedance for odd-mode configuration for t = 0,

$$Z_{Oo} = (\eta/\sqrt{\epsilon_{reff}}) \; [1/\{(w/h)[1 + (1/3\sqrt{\epsilon_{reff}})] + (4/3\sqrt{\epsilon_{reff}})(1/(s/w) + 1)\}]$$

3.3 PHASE VELOCITY FOR EVEN AND ODD-MODES

The phase velocity can be calculated by the formula

$$V_P = c/\sqrt{\epsilon_{reff}}$$

In case of coupled microstripline structure there are two modes of propagation even and odd-modes.

For even-mode, $\qquad V_{Pe} = c/(\sqrt{\epsilon_{reff}})_e$

For odd-mode, $\qquad V_{Po} = c/(\sqrt{\epsilon_{reff}})_o$

3.4 GUIDE WAVE LENGTH FOR EVEN AND ODD-MODES

The guide wave length also determined the characteristic parameters of the transmission structure and is the functions of strip geometry permittivity and operating frequency for TM-mode of propagation for low frequency guide wave length, written as:

$$\lambda_g = V_P/f = \lambda_o/\sqrt{\epsilon_{reff}}$$

For even-mode propagation, $\qquad \lambda_{ge} = V_P/f = \lambda_o/(\sqrt{\epsilon_{reff}})_e$

For odd-mode propagation, $\qquad \lambda_{go} = V_P/f = \lambda_o/(\sqrt{\epsilon_{reff}})_o$

4. COMPUTATION OF THE RESULT

For the study of characteristic impedance, phase velocity and guide wave length of the microstripline coupler and their variation with strip geometries, substrate permittivity and frequency exhaustive computations have been carried out with the help of computer aided programme developed by S.K. Kaul, CARE, DELHI.

The results obtained have been placed in tabular forms.

Exhaustive computations have been carried out by putting the different variables and fixed parameters as given in the following sections:

(i) Dependence of characteristic impedance for even and odd-mode on stripwidth of the metal.

(ii) Dependence of characteristic impedance for even and odd-mode on spacing between two metal strips.

(iii) Dependence of characteristic impedance for even and odd-mode on dielectric constant of the substrate material.

(iv) Dependence of phase velocity and guide wave length on stripwidth for even and odd-modes.

(v) Dependence of phase velocity and guide wave length on dielectric constant of the substrate material for even and odd-modes.

Within the permissible range of frequency in which TEM-mode of propagation holds well. The results have been placed in different tables and corresponding graphs have been drawn.

ANALYTICAL STUDY OF MICROSTRIPLINE COUPLER

Table No.- 01

Dependence of characteristic impedance on width of the metal strip taking spacing as parameter

h = 100 mils, f = 2GHz, t = 0.05 mils, ϵ_r = 9.6

1 mil = 2.54 x 10-3 cm

W(mils)	S = 10 mils		S = 20 mils		S = 50 mils	
	$Z_{oe}(\Omega)$	$Z_{oo}(\Omega)$	$Z_{oe}(\Omega)$	$Z_{oo}(\Omega)$	$Z_{oe}(\Omega)$	$Z_{oo}(\Omega)$
10	166.50	60.20	153.40	61.50	134.80	78.60
30	120.30	41.80	116.20	52.10	101.40	59.50
50	98.50	37.60	92.40	46.80	87.70	52.10
70	84.10	34.20	87.40	37.70	75.80	47.50
90	74.20	32.10	72.60	35.90	68.30	42.30

ANALYTICAL STUDY OF MICROSTRIPLINE COUPLER

Graph No.- 01

Dependence of characteristic impedance on width of the metal strip taking spacing as parameter
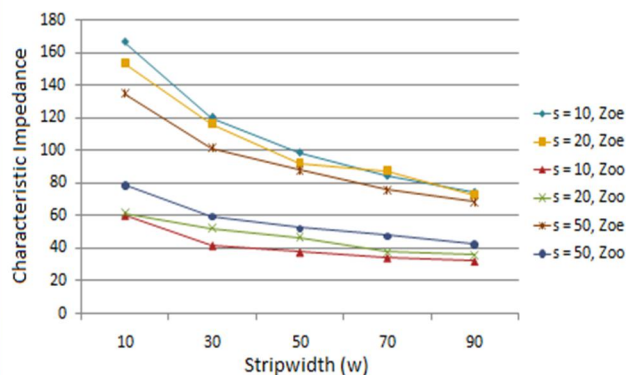
ANALYTICAL STUDY OF MICROSTRIPLINE COUPLER

Table No.- 03

Dependence of characteristic impedance on dielectric constant

w = 100 mils, h = 100 mils , t = 0.05 mils, s = 100 mils, f = 2GHz

ϵ_r	Z_{oe}	Z_{oo}
2.5	107.90	76.80
9.6	62.40	45.20
16.0	47.50	36.30
18.0	43.20	34.10

ANALYTICAL STUDY OF MICROSTRIPLINE COUPLER

Table No.- 02

Dependence of characteristic impedance on spacing between the metal strips taking width as parameter

h = 100 mils, f = 2GHz, t = 0.05 mils, ϵ_r = 9.6

1 mil = 2.54 x 10-3 cm

s (mils)	w = 10 mils		w = 30 mils		w = 70 mils	
	Z_{oe} (Ω)	Z_{oo} (Ω)	Z_{oe} (Ω)	Z_{oo} (Ω)	Z_{oe} (Ω)	Z_{oo} (Ω)
10	166.50	60.40	120.30	41.80	84.10	34.20
20	153.40	61.50	116.20	50.10	80.40	37.70
50	134.80	78.60	101.40	59.50	75.80	47.50
100	122.30	95.20	92.20	70.30	71.50	53.20

ANALYTICAL STUDY OF MICROSTRIPLINE COUPLER
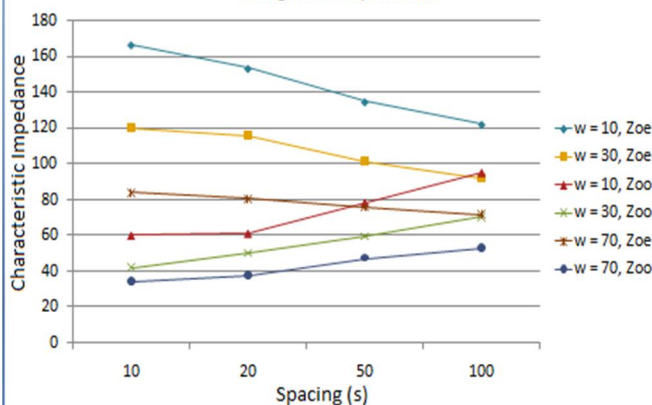
Graph No.- 03

Dependence of characteristic impedance on dielectric constant
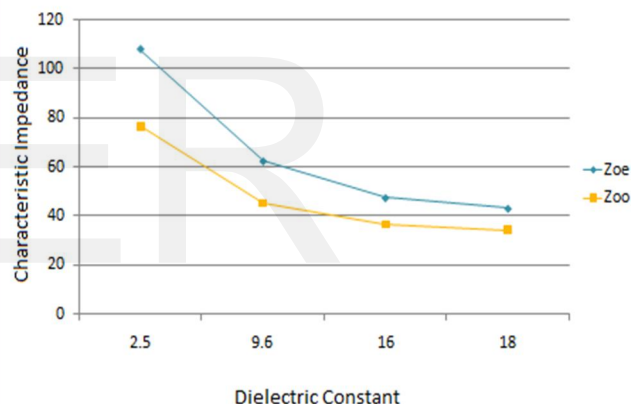
ANALYTICAL STUDY OF MICROSTRIPLINE COUPLER

Graph No.- 02

Dependence of characteristic impedance on spacing between the metal strips taking width as parameter

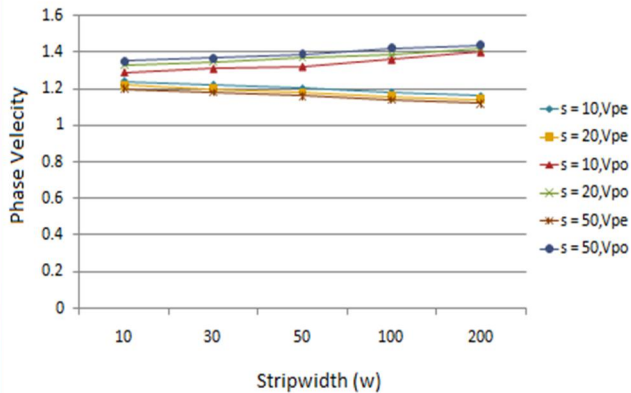## ANALYTICAL STUDY OF MICROSTRIPLINE COUPLER

Table No.- 04

Dependence of phase velocity on stripwidth keeping spacing as a parameter

h = 100 mils, f = 3GHz, t = 0.01 mils, ϵ_r = 10

w (mils)	s = 10 mils		s = 20 mils		s = 50 mils	
	Vpe x 10⁸ m/s	Vpo x 10⁸ m/s	Vpe x 10⁸ m/s	Vpo x 10⁸ m/s	Vpe x 10⁸ m/s	Vpo x 10⁸ m/s
10	1.24	1.29	1.22	1.33	1.20	1.35
30	1.22	1.31	1.20	1.35	1.18	1.37
50	1.20	1.32	1.18	1.37	1.16	1.39
100	1.18	1.36	1.16	1.39	1.14	1.42
200	1.16	1.40	1.14	1.42	1.12	1.44

ANALYTICAL STUDY OF MICROSTRIPLINE COUPLER

Graph No.- 04

Dependence of phase velocity on stripwidth keeping spacing as a parameter

ANALYTICAL STUDY OF MICROSTRIPLINE COUPLER

Table No.- 06

Dependence of phase velocity & guide wavelength on dielectric constant

h = 100 mils, s = 100 mils, f = 2GHz, w = 100 mils

ϵ_r	$V_{pe} \times 10^8$ m/s	$V_{po} \times 10^8$ m/s	λ_{ge} (cm)	λ_{go} (cm)
2.5	2.12	2.22	10.6	11.10
9.6	1.11	1.26	5.55	6.30
16	0.86	0.99	4.30	4.95
18	0.81	0.93	4.05	4.65

ANALYTICAL STUDY OF MICROSTRIPLINE COUPLER
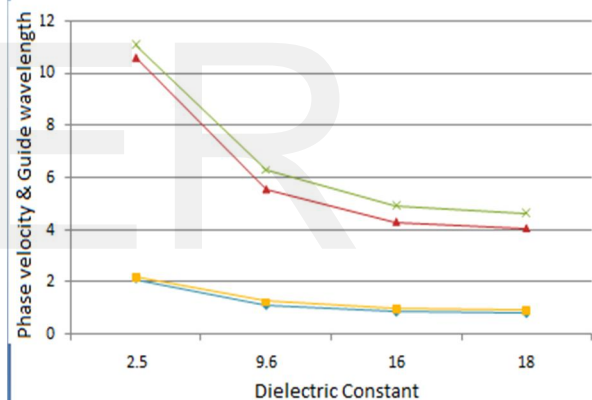
Table No.- 05

Dependence of guide wavelength on stripwidth keeping spacing as a parameter

h = 100 mils, f = 2GHz, t = 0.05 mils, ϵ_r = 9.6

w (mils)	s = 10 mils		s = 20 mils		s = 50 mils	
	λ_{ge}(cm)	λ_{go} (cm)	λ_{ge}(cm)	λ_{go} (cm)	λ_{ge}(cm)	λ_{go}(cm)
10	6.2	6.45	6.1	6.65	6.00	6.75
50	6.1	6.55	6.0	6.75	5.90	6.85
100	6.0	6.60	5.9	6.85	5.80	6.95
150	5.9	6.80	5.8	6.90	5.70	7.10
200	5.8	7.0	5.7	7.10	5.60	7.20

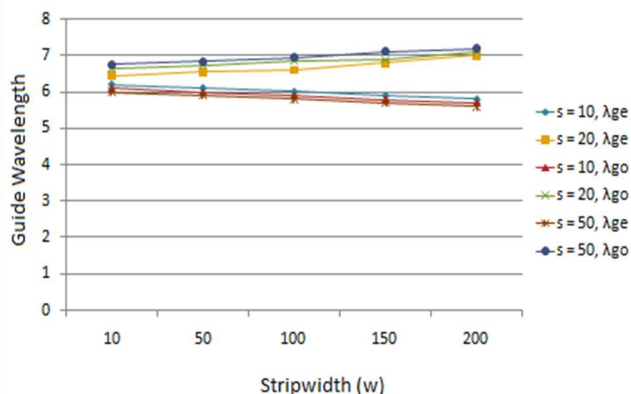## ANALYTICAL STUDY OF MICROSTRIPLINE COUPLER

Graph No.- 06

Dependence of phase velocity & guide wavelength on dielectric constant

ANALYTICAL STUDY OF MICROSTRIPLINE COUPLER

Graph No.- 05

Dependence of guide wavelength on stripwidth keeping spacing as a parameter

5. DISCUSSION AND CONCLUSION OF THE RESULT

The above study reveals that characteristic impedance for even and odd-modes decreases with stripwidth.

With increase of spacing between two metal strips, the characteristic impedance decreases in case of even-mode and increases in case of odd-mode. That means when metal strips are widely separated more and more flux lines and power are concentrated in case of even-mode and lesser flux lines and power concentrated in case of odd-mode.

Result also showed that the phase velocity and guide wavelength decreases with increase of width of metal strips in case of even-mode and both parameters increases with increase of spacing between two metal strips. Also these parameters for even-mode are smaller than those of odd-modes.

The result discussed above provides useful guidelines for the design of different microstripline structures such as: Coupler, Directional Coupler, Isolator, Circulators etc.

These results are also very useful for the study of reflection and transmission co-efficient of the microstripline coupler in case of both even and odd-modes of propagation.

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